127: Philosophical Logic
Sample Paper

HT15

Answer THREE questions.

1. (a) Show, using induction, that Kleene's three-valued semantics for propositional logic, PL, results in there being no Kleene-valid PL-wffs. [6]

   (b) Suppose we kept the Kleene truth-tables, but treated both 1 and # as designated values. Would there still be no Kleene-valid PL-wffs? Justify your answer. [3]

   (c) Show that in Łukasiewicz's system $\sim(P \to Q) \vDash_{Ł} \sim(P \to (P \to Q))$ but $\nvDash_{Ł} \sim(P \to Q) \to \sim(P \to (P \to Q))$. [6]

   (d) Are there any examples in Kleene's system that have a similar feature: namely, there exist two PL-wffs ϕ and ψ such that $\phi \vDash_K \psi$ but $\nvDash_K (\phi \to \psi)$? Justify your answer. [2]

   (e) Discuss whether it is a problematic feature for any logic to have two wffs ϕ and ψ such that $\phi \vDash \psi$ but $\nvDash (\phi \to \psi)$ [8]

2. Let vW be the following axiomatic system for the language of modal propositional logic, MPL.

   - *Rules*: MP, plus the following:

$$\frac{\phi \leftrightarrow \psi}{\Box\phi \leftrightarrow \Box\psi} \tag{R1}$$

   - *Axioms*: all instances of PL1–PL3, plus the following:

$$\Box \sim(\phi \wedge \sim \phi) \tag{A1}$$

$$\sim \Box(\phi \wedge \sim \phi) \tag{A2}$$

$$\Box(\phi \wedge \psi) \leftrightarrow (\Box\phi \wedge \Box\psi) \tag{A3}$$

   (a) Glossing $\Box\phi$ as "it is obligatory that ϕ", provide idiomatic English glosses of (R1) and (A1)–(A3). In each case, comment on the plausibility of the resulting English inference or principle. [8]

   (b) Show that:

      (i) $\vdash_{\mathsf{vW}} \Box\phi \to \sim \Box \sim \phi$

      (ii) $\vdash_{\mathsf{vW}} \Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$

      [*Hint: $\phi \to \psi$ is provably equivalent to $\phi \leftrightarrow (\phi \wedge \psi)$ in propositional logic*]

      (iii) if $\vdash_{\mathsf{vW}} \phi$, then $\vdash_{\mathsf{vW}} \Box\phi$ [12]

      [*In each case, propositional steps may be abbreviated as usual.*]

   (c) Hence, or otherwise, show that each D-valid wff is a vW-theorem.

      [*You may assume D-completeness without proof.*] [5]

3. (a) Specify the syntax for the language of Stalnaker's conditional, SC. [2]

   (b) State necessary and sufficient conditions for a triple $\langle \mathscr{W}, \preceq, \mathscr{I} \rangle$ to be an SC-model; explain how these conditions are modified to obtain the definition of an LC-model, for Lewis's conditional. [6]

   (c) Formalize the following argument in the language of SC so that its conclusion is an SC-semantic consequence of its premisses, and provide a semantic argument to show that this is indeed the case. Comment on any difficulties or points of interest. [12]

   > If Brown had become PM in 2010, there would have been a rainbow coalition. For it's unimaginable and quite impossible that Conservative and Labour MPs should ever sit side-by-side on the government benches. Moreover, had Brown become PM, Cameron would have resigned and either Conservatives would have sat alongside Labour MPs on the government benches or there would have been a rainbow coalition or Brown would have led a minority government. It's not the case that had Cameron resigned, Brown would have led a minority government. And, if Cameron had gone, Brown most certainly would have become PM.

   (d) Is the conclusion of the formalized argument also an LC-semantic consequence of its premisses? Justify your answer. [5]

4. (a) For each of the following wffs in the language of the predicate calculus, PC, with the symbol ι for definite descriptions, specify a semantically equivalent wff in the language of PC without ι.

   (i) $\exists x(Bx \wedge A\iota xDx)$ [2]

   (ii) $\forall x(Cax \rightarrow B\iota xCx\iota yDy)$ [2]

   (b) Assume an English sentence has been correctly formalized as $\forall x\,(Px \rightarrow Qfx)$ in the language of PC with function symbols. How can the sentence be correctly formalized in the language of PC with identity (but without function symbols or ι)? [2]

   (c) Discuss the best ways to formalize the following sentence:

   Sherlock Holmes owns three deerstalker caps

   (i) in PC with ι and identity and (ii) in Sider's version of free logic, FPC, with identity. Make sure you give a full formalization of *owns three deerstalker caps*. Discuss the advantages and problems of each formalization. [6]

   (d) Determine whether or not the following claims are correct. Substantiate your answers.

   (i) $\vDash_{\text{FPC}} \forall xPx \rightarrow Pa$ [2]

   (ii) $\vDash_{\text{FPC}} Pa \rightarrow \exists x\, x = a$ [2]

   (iii) $\vDash_{\text{FPC}} \forall y(\forall xPx \rightarrow Py)$ [2]

   (e) Show that the following holds for each wff ϕ in the language of PC with identity (but without function symbols or ι): if $\vDash_{\text{FPC}} \phi$ then $\vDash_{\text{PC}} \phi$ [7]

5. (a) A PC-model $\langle \mathscr{D}, \mathscr{I} \rangle$ is said to be a model of a set S of PC-wffs iff all members of S are true in $\langle \mathscr{D}, \mathscr{I} \rangle$. Provide models for each of the following sets:

(i) $\{\forall x\, \exists y\, \exists z\, (Rxy \wedge Rxz \wedge Ryz), \sim \exists x\, Rxx, \forall x\, \forall y\, \forall z\, (Rxy \to (Ryz \to Rxy))\}$ [3]

(ii) $\{\exists x \sim Px\} \cup \{P\alpha :\ \alpha$ is an individual constant$\} \cup \{\sim \alpha = \beta :$ α and β are distinct individual constants$\}$ [3]

(b) Using only a two-place predicate R for 'x is an element of y' formalize the following statement: [3]

For every set there is a set that contains all its subsets.

(c) Assume that P and Q are one-place predicates and that the wff ψ is obtained from the wff ϕ by substituting all occurrences of P with Q. Prove by induction on the construction of ψ that $\{\forall x\, (Px \leftrightarrow Qx), \phi\} \vDash_{\mathrm{PC}} \psi$. [8]

(d) State without proof the Compactness Theorem for PC, as formulated in Sider's book. Deduce that if $\Gamma \vDash_{\mathrm{PC}} \phi$, then there is a finite subset Δ of Γ such that $\Delta \vDash_{\mathrm{PC}} \phi$. [3]

(e) Consider the following sentence:

Some computers communicate only with one another.

How may one formalize this sentence in (i) the language of PC with identity and (ii) second-order logic? Discuss the advantages and disadvantages of your formalizations. [5]

6. Consider the following wffs of the language of quantified modal logic, QML:

$$\forall x \Box \exists y (y = x) \qquad (\alpha)$$

$$\Diamond \exists x \exists y \forall z (z = x \vee z = y) \to \Box \exists x \exists y \forall z (z = x \vee z = y) \qquad (\beta)$$

(a) Translate α and β into idiomatic English (reading $\Box$ as metaphysical necessity). [4]

(b) (i) Define what it takes for a triple $\langle \mathscr{W}, \mathscr{D}, \mathscr{I} \rangle$ to be a SQML-model.

(ii) Show that $\vDash_{\mathrm{SQML}} \alpha$ and $\vDash_{\mathrm{SQML}} \beta$. [7]

(c) (i) Define what it takes for a 5-tuple $\langle \mathscr{W}, \mathscr{R}, \mathscr{D}, \mathscr{Q}, \mathscr{I} \rangle$ to be a VDQML-model.

(ii) Show that $\nvDash_{\mathrm{VDQML}} \alpha$ and $\nvDash_{\mathrm{VDQML}} \beta$. [7]

(d) Discuss the philosophical significance, if any, of these results. [7]

7. Let $\mathcal{L}_{2D}$ be the language of quantified modal logic, QML, enriched with the operators $@$, $\times$, F, $\mathsf{F}@$ and $\mathsf{F}\Box$.

   (a) Formalize the following sentences in $\mathcal{L}_{2D}$. Discuss any difficulties, ambiguities and points of interest.

      (i) It isn't possible that all professors who are actually logicians could have become moral philosophers. [3]

      (ii) It's possible that if Hannibal had won the Second Punic War then all those who would then actually speak some semitic language might also speak some Germanic language. [3]

   (b) Explain what it means for a formula ϕ of $\mathcal{L}_{2D}$ to be *superficially necessary* in a model $\mathscr{M}$ and what it means for the formula to be *deeply necessary* in $\mathscr{M}$. [2]

   (c) Consider the following wff:

$$\exists x\,(Tx \wedge \forall y\,(Ty \rightarrow x = y)) \rightarrow \exists x\,(@Tx \;\wedge\; \forall y\,(@Ty \rightarrow x = y) \wedge Tx) \qquad (\gamma)$$

      (i) Show that γ is 2D-valid. [4]

      (ii) Specify a model in which γ is superficially contingent. Sketch a proof demonstrating that γ is superficially contingent in your model. [4]

   (d) The wff γ is proposed as a formalization of the following English sentence:

      If anyone singlehandedly invented the telephone, the actual inventor of the telephone invented the telephone.

   Discuss the philosophical significance, if any, of the results in (c) when this English sentence is formalized in this way. [9]